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A STUDY TO ANALYSE THE QUALITY OF PROBLEMS POSED BY PRE-SERVICE MATHEMATICS TEACHERS ON SQUARE ROOT EXPRESSIONS

UM ESTUDO PARA ANALISAR A QUALIDADE DOS PROBLEMAS COLOCADOS POR PROFESSORES DE MATEMÁTICA EM FORMAÇÃO EM EXPRESSÕES DE RAIZ QUADRADA

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ABSTRACT

In this study, it was aimed to examine the problems that the pre-service mathematics teachers posed about the square root expressions. 35 pre-service teachers studying in the third grade of the middle school mathematics teaching department of a state university participated in this study in which the case study method was used. Each pre-service teacher was asked to pose 2 free problems related to the square root expressions. The collected data were analyzed according to the framework created by the researcher in line with the literature. Accordingly, the problems that the pre-service mathematics teachers posed were examined in terms of mathematically, data quality, grammar and expression, instructions and amount of data in the problems, solvability and overall assessment (availability). As a result, it was seen that pre-service teachers generally set up problems involving daily life, more than half of the problems they set up were solvable, but there were also problems that could not be solved due to logic errors, conceptual errors and lack of mathematical instructions. In addition, although there were errors in terms of language and expression in the problems, the majority of the pre-service teachers used language and expression at an adequate level.

Keywords: problem posing, pre-service mathematics teachers, square root expressions.

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RESUMO

Neste estudo, objetivou-se examinar os problemas que os futuros professores de matemática colocaram sobre as expressões de raiz quadrada. Participaram deste estudo 35 futuros professores do terceiro ano do departamento de ensino de matemática do ensino fundamental de uma universidade estadual, no qual foi utilizado o método de estudo de caso. Cada professor em formação foi solicitado a propor 2 problemas livres relacionados às expressões de raiz quadrada. Os dados coletados foram analisados conforme referencial criado pela pesquisadora em consonância com a literatura. Assim, os problemas que os futuros professores de matemática colocaram foram examinados em termos matemáticos, qualidade dos dados, gramática e expressão, instruções e quantidade de dados nos problemas, solubilidade e avaliação global (disponibilidade). Como resultado, viu-se que os futuros professores geralmente montavam problemas envolvendo o cotidiano, mais da metade dos problemas que eles armavam eram solucionáveis, mas também havia problemas que não podiam ser resolvidos por erros de lógica, erros conceituais e falta de instruções matemáticas. Além disso, embora houvesse erros em termos de linguagem e expressão nos problemas, a maioria dos futuros professores utilizou a linguagem e a expressão em um nível adequado.

Palavras-chave: formulação de problemas, professores de matemática em formação, expressões de raiz quadrada.

Introduction

Mathematics education and training are designed to facilitate the acquisition of mathematics-specific knowledge and skills by students for their practical use in both daily life and academic contexts (Ev-Cimen & Yildiz, 2017). Among these proficiencies, problem-posing stands out as a crucial competence. It encompasses various cognitive skills, including problem-solving abilities, associative thinking, the effective utilization of mathematical language, and the cultivation of critical thinking. These competencies are integral components of the curricula adopted by many nations. For instance, our national mathematics curriculum (Ministry of National Education, [MoNE], 2013) incorporates problem posing as a complementary skill to problem-solving techniques. Similarly, in the Australian national communiqué, is intertwined with the development of critical thinking and reasoning (Australian Education Council and Curriculum Corporation, 1991).

The prominence of problem-posing skills, previously overshadowed by the emphasis on problem solving within curricula, has witnessed a resurgence in recent years due to educational reforms and comprehensive studies (Chinese Ministry of Education, 2011; National Council of Mathematics Teachers [NCTM], 2000). This transformation has been motivated by the recognition of limitations in existing

textbooks, which often fail to account for the diverse characteristics of students. Consequently, the belief has arisen that the reinterpretation and adaptation of mathematical problems to suit individual student characteristics could yield multifaceted advantages for the teaching and learning of mathematics (Cai et al., 2015; Akay et al., 2006; Cifarelli & Sevim, 2015; English, 2020; Turhan & Güven, 2014)."

Problem posing, its place in teaching and its importance for pre-service teachers

Problem posing, as defined by various scholars (Duncker, 1945; NCTM, 2000; Silver, 1994), refers to the process of generating novel problems derived from existing data related to a pre-existing problem, situations, and events encountered in everyday life (Ambrus, 1997; Stickles, 2006), or one's own personal experiences (Stoyanova & Ellerton, 1996). Lavy and Bershadsky (2002) underscore the significance of problem posing in mathematics education, asserting that it represents a more advanced cognitive activity than problem solving, facilitating the cultivation of creativity. Moreover, problem posing is found to be advantageous for students in terms of enhancing their comprehension, attitudes, autonomy, and reasoning skills in the context of mathematics (Brown & Walter, 1983; Burton, 1999; Matz & Leizer, 1992; Silver & Cai, 1996). In line with this perspective, Kojima, Miwa, and Matsui (2009) contend that during the process of problem posing, students engage in diverse modes of thinking, which, in turn, fosters the development of a profound cognitive structure.

The integration of problem posing into teaching practices offers students the opportunity to generate alternative content compared to conventional problem-solving tasks, which often involve the repetition of prescribed solutions and standard textbook problems that do not cater to individual differences. This approach not only introduces flexibility into teaching but also benefits both students and educators (Akay, 2006; Harskamp & Suhre, 2006; Ev-Cimen & Yildiz, 2017). As a result, teachers can engage students as active participants in the teaching and learning process, as suggested by scholars (Baykul, 1999; Olkun & Toluk, 2003). Furthermore, the ability to formulate problems based on personal experiences and

knowledge within the context of real-life scenarios enables students to recognize and address challenges they encounter in their daily lives, fostering a tangible connection between the curriculum and real-world applications (Turhan & Güven, 2014). Considering the advantages of problem posing from an instructor's perspective, it can serve as a valuable tool to enhance the quality of teaching, assessment, and deepen students' comprehension (Lavy & Shriki, 2007; Lin, 2004; Tekin-Sitrava & Isik, 2018).

Taking into consideration the influential role of teachers as role models, the incorporation of a wide array of diverse problem scenarios enables students to cultivate multifaceted perspectives on the subject matter (Işık, Işık, & Kar, 2011). Teachers' proactive efforts to guide students in articulating their thoughts using appropriate mathematical language through the analysis of real-life situations not only facilitate students' growth in this domain but also furnish teachers with valuable insights into students' misconceptions and prior subject knowledge (Lin & Leng, 2008; Silver & Cai, 2005). Tailoring the quality of the questions used in lessons based on the information gathered about students' comprehension enhances the overall quality of instruction (Hidayati & Kuniasari, 2021).

An essential determinant for elevating the standard of teaching and realizing these potential benefits for both students and the educational environment is the teacher's knowledge and experience (Crespo, 2015; Kılıç, 2017; NCTM, 2000). The teacher's expertise in this domain is linked to their professional background, the diversity of students they encounter, and the quality of their pre-service teacher training (Sayın & Orbay, 2023; Ulusoy & Kepceoğlu, 2018). An examination of the problem-posing skills of pre-service teachers, especially regarding complex mathematical concepts like radical expressions, which have a significant historical impact in the realm of mathematics, may offer valuable insights into the effectiveness of pre-service teacher education (Korkmaz & Gür, 2006). Consequently, the findings from such investigations have the potential to contribute to the improvement of programs like in-service training, undergraduate education, and internships, which provide practical experience to both teachers and pre-

service teachers (Crespo & Sinclair, 2008; Işık, Kar, Yalçın, & Zehir, 2011; Stickles, 2006).

Square root expressions and problem posing

In the history of humanity, numbers have played a major role in making sense of nature and meeting the necessary needs. While natural numbers, integers and rational numbers were discovered earlier than irrational numbers in history, it took a long time to discover and accept the existence of irrational numbers. The reason for this is that irrational numbers have no equivalent in nature due to their structure and it is difficult to represent them on the number line. These difficulties in irrational numbers also cause difficulties in discovering and learning the set of real numbers, which is a combination of irrational and rational numbers (Güler, 2017).

In the Mathematics Curriculum (2018), Square Root Expressions is a topic that takes place in the process of creating the set of real numbers by revealing the existence of the set of irrational numbers, which are non-rational numbers, after learning natural numbers, integers and rational numbers. In the Mathematics Curriculum (2018), there are 52 learning outcomes. 8 of these objectives include the topic of square root expressions. This corresponds to 15% of all objectives. The topic of square root expressions is included for the first time in the 8th grade and is among the topics that students frequently encounter in mathematics courses during secondary education. In addition, it also forms the basis of more complex mathematical topics in high school. In this sense, learning square roots correctly is one of the basic steps to increase mathematical success in high school and to cope with more complex topics more easily (Kaplan, Altaylı & Öztürk, 2014). The topic of square root expressions is used in different disciplines and subject areas. In addition to the fact that the subject is complex and abstract, the fact that it has little relation with daily life causes it to be among the most difficult subjects for students (Dinç & Yenilmez, 2022; Ercire, Narlı & Aksoy, 2016; Shınno, 2018; Yılmaz & Güzel, 2020).

One of the goals of mathematics education is to educate students who have developed problem-solving skills in order to overcome the difficulties experienced by students (Katrancı & Şengül, 2019). Kopparla et al. (2018) emphasized that the

problems to be presented by teachers should enable students to reason, bring evidence to their thinking, communicate by sharing their mathematical ideas, and make connections between mathematics and real life. Therefore, teachers should be able to formulate and pose valuable problems for their students because effective problem posing is critical for high-quality mathematics instruction (Cai & Hwang, 2020). In this sense, training teachers with basic knowledge and skills in problem posing has an important place in mathematics education (Sawhney & Bansal, 2015). The inclusion of problem posing in teacher education contributes to the development of subject matter and pedagogical content knowledge (Silver, 1994; Voica & Singer, 2020). For this reason, many mathematics educators have recommended the inclusion of problem posing during teacher education to create more opportunities for pre-service teachers to experience process learning that will influence their future classroom pedagogical strategies (Zhang & Cai, 2021; Rosli, Capraro, & Capraro, 2014). Considering the role of teachers, it is necessary to first evaluate the competence of pre-service teachers in the process of becoming a teacher. In this context, considering the nature of square root expressions and the importance of problem posing skills, this study aimed to examine the free problem posing skills of prospective elementary mathematics teachers about square root expressions.

Methodology

In the course of this investigation, a comprehensive analysis was conducted on the nature of problems posed by pre-service mathematics teachers concerning square root expressions. To facilitate this examination, a qualitative research approach known as the case study method was employed (Creswell & Creswell, 2017). The case study method is a research approach that enables an in-depth exploration of an event, activity, process, or phenomenon, involving one or more individuals, with a specific focus on addressing "how" and "why" inquiries (Yıldırım & Şimşek, 2013).

Participants

This research involved the participation of 35 pre-service mathematics teachers who were enrolled at a state university in the western region of Turkey. Among these participants, 27 were in their third year of study and identified as female, while the remaining eight were male. The selection criterion for these individuals was their prior completion of the "teaching of numbers" course. Furthermore, it is essential to emphasize that the principle of voluntary participation was strictly adhered to in this study.

Data Collection Tools

In this study, the "free problem pose form about square root expressions" created by the researchers was used as a data collection tool. In this form, pre-service mathematics teachers were asked to pose two free problems about square root expressions. The question in the form is as follows:

1. Pose two free problems at the middle school level using square root expressions.

This form was sent to the pre-service mathematics teachers online and the answers were received online.

Analysis of the Data

Qualitative analysis methods were used to analyze the data collected in this study. The problem posing evaluation criteria created by Yıldız and Özdemir (2015) were taken into consideration in the analysis of the problems posed by pre-service mathematics teachers about square root expressions. According to the framework in Table 1, the problems were analyzed in six categories: mathematicality, data quality, grammar and expression, instructions and amount of data in the problems, solvability and overall assessment (availability).

Table 1 – Problem-Posing Evaluation Criteria (Yıldız &Özdemir, 2015)

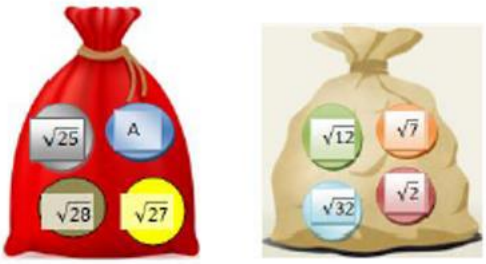
Evaluation Criteria	Subcomponents of the Criteria
Mathematicality	Use of mathematical expressions
	Use of mathematical concepts
	Use of mathematical symbols and notations, if any
	If the problem is visual (figure, table, graph, etc.), the transfer between text and image.
Data quality	Logical and mathematical appropriateness of the data in the problem
	The significance of the result that can be reached with the data in the problem
	If the problem requires the use of a unit, the existence and expression of the unit
Grammar and Expression	Compliance of the question text with the grammar rules
	Whether it contains expression disorder or spelling mistakes
	The use of punctuation marks
Instructions and Amount of Data in the Posed Problem	Expressions used when posing the problem or guiding the process to be done with the problem
	Appropriateness of the steps or conditions specified in the instructions used in the problem
	The amount of data contained in the problem in order to be able to solve the problem
Solvability	Achievability of the desired outcome of the problem
Overall Assessment (Availability)	Usability of the problem in the teaching process at middle school level

The problems posed by the pre-service teachers were analyzed separately for each criterion. In line with the sub-evaluation criteria, the extent to which they met each evaluation criterion was expressed as points and percentages. Since a total of 70 problems were posed by the pre-service mathematics teachers, the evaluation of these criteria was based on the total number of questions. For example, since the sub-criterion of the mathematicality criterion, "the use of mathematical

expressions", was met in 65 out of 70 problems, the score obtained from this sub-criterion was 65 out of 70 and the success rate was determined as 92.8%.

Table 2 shows an example analysis of a problem posed by one of the pre-service mathematics teachers.

Problem:



Kırmızı torba ve sarı torbada toplar ve topların üzerinde rakamlar yazmaktadır. İki torbadaki topların üzerindeki sayılar çarpılıp sonuçlar yazılıyor. Çarpımların doğal sayı olma olasılığı 1/8 ise A kaç olabilir?

The problem is translated as follows.

"There are balls in red and yellow bags and figures on the balls. The figures on the balls in the two bags are multiplied and the results are written. If the probability of the products being natural numbers is 1/8, what can A be?"

Table 2 – Analysis of a problem posed by pre-service mathematics teachers according to evaluation criteria

Evaluation Criteria	Subcomponents of the Criteria	Descriptions
Mathematicality	Use of mathematical expressions	Square Root Expressions were used correctly.
	Use of mathematical concepts	The term of "figure" has been misused
	Use of mathematical symbols and notations, if any	The square root symbol was used correctly.
	If the problem is visual (figure, table, graph, etc.), the transfer between text and image.	The visual created and the problem posed are in harmony.
Data quality	Logical and mathematical appropriateness of the data in the problem	The data are in logical harmony.
	The significance of the result that can be reached with the data in the problem	The result to be reached is meaningful.

	If the problem requires the use of a unit, the existence and expression of the unit	The use of units is not required.
Grammar and Expression	Compliance of the question text with the grammar rules	Complies with grammar rules.
	Whether it contains expression disorder or spelling mistakes	There is a minor expression disorder.
	The use of punctuation marks	Punctuation marks are used.
Instructions and Amount of Data in the Posed Problem	Expressions used when posing the problem or guiding the process to be done with the problem	The instruction is not clear. It is not specified whether the numbers to be multiplied are to be selected from the same bag or from opposite bags.
	Appropriateness of the steps or conditions specified in the instructions used in the problem	Since the instruction was not clear, the steps specified were not deemed appropriate.
	The amount of data contained in the problem in order to be able to solve the problem	There is sufficient data.
Solvability	Achievability of the desired outcome of the problem	It was not found solvable due to mathematical, data quality and instruction errors.
Overall Assessment (Availability)	Usability of the problem in the teaching process at middle school level	Since it was not solvable, it was evaluated as not usable at the middle school level.

Validity and Reliability

The problems posed by pre-service mathematics teachers about the concept of square root expressions were scored separately by three researchers according to the problem posing evaluation criteria. Afterwards, the differences between the scoring of the three researchers were reviewed and a common opinion was reached.

In addition, reliability was tried to be increased by analyzing some of the problems posed by the pre-service mathematics teachers according to the evaluation criteria.

Results

In this study, it was aimed to examine pre-service mathematics teachers' ability to pose problems about square root expressions at the middle school level and the qualities of these problems. In addition, the percentage of the problems that met the determined sub-criteria was analyzed.

The problems posed by the pre-service mathematics teachers were examined in terms of 6 sub-criteria (Mathematicality, Data quality, Grammar and expression, Instructions and amount of data, Solvability and General evaluation) in the scoring instructions.

Table 3 – Analysis of Problems According to Mathematicality Criteria

Criteria and Sub-Criteria	Point	Percent
Mathematics	236	84.2
1.Use of mathematical expressions	65	92.8
2.Use of mathematical concepts	55	78.5
3.Use of mathematical symbols and notations, if any	63	90
4.If the problem is visual (figure, table, graph, etc.), the transfer between text and image.	42	60

The 70 problems related to square root expressions posed by pre-service mathematics teachers were first analyzed according to the mathematicality criterion. Since each problem was examined according to the sub-criteria of the mathematicality criterion, 1 point was given for each criterion provided. Therefore, a maximum of 280 points can be obtained from the mathematicality criterion. According to Table 3, 236 points were obtained from the mathematicality criterion and 84% success rate was achieved. The most successful sub-criterion was the use of mathematical expressions with a rate of 92.8%. As a matter of fact, mathematical expressions were used correctly in 65 of the 70 problems. While a success rate of 90% was achieved in the criterion of using mathematical symbols and notation, it

was determined that there were errors in the transfer between text and visualization and the success rate was calculated as 60%.

Table 4 – Analysis of Problems According to Data Quality Criteria

Criteria and Sub-Criteria	Point	Percent
Data Quality	153	72.8
1.Logical and mathematical appropriateness of the data in the problem	49	70
2.The significance of the result that can be reached with the data in the problem	49	70
3.If the problem requires the use of a unit, the existence and expression of the unit	55	78.5

The results of the analysis of the problems related to square root expressions according to the data quality category are given in Table 4. The maximum score that can be obtained for data quality is 210. Of the 70 problems posed by the pre-service mathematics teachers, 153 points were obtained for the data quality criterion and the success of meeting this criterion was calculated as 72.8%. However, 49 points were obtained from the category of "logical and mathematical appropriateness of the data in the problem" and it was determined that 70% of this sub-criterion was met. In relation to this sub-criterion, that is, if the data is logically appropriate, it was seen that 70% success was achieved in the sub-criterion of "the meaningfulness of the result that can be reached with the data in the problem". In addition, it was concluded that mathematical units were used and expressed correctly in 55 out of 70 problems.

Table 5 – A nalysis of Problems According to Grammar and Expression Criteria

Criteria and Sub-Criteria	Point	Percent
Grammar and Expression	144	68.5
Compliance of the question text with the grammar rules	49	70
Whether it contains expression disorder or spelling mistakes	48	68.5
The use of punctuation marks	47	67.1

The analysis of the problems related to square root expressions according to the grammar and expression criteria is given in Table 5. The maximum score that can be obtained from this criterion is 210. When Table 2 is analyzed, it is seen that

68.5% of the grammar and expression criterion is met in the posed problems. It was determined that the part where the pre-service mathematics teachers made the most mistakes was grammar and expression. As a matter of fact, it was determined that 21 of the 70 problems they posed were deficient in grammar, 22 were deficient in spelling mistakes and expression disorders, and 23 were deficient in incomplete use of punctuation marks. Success rates of 70%, 68.5% and 67.1% were achieved in these sub-criteria, respectively, and it stood out as the category with the highest failure rate.

Table 6 – Analyses of the Problems According to the Criteria of Instructions and Data Amount in the Posed Problem

Criteria and Sub-Criteria	Point	Percent
The Instructions and the Amount of Data in the Posed Problem	189	90
1.Expressions used when posing the problem or guiding the process to be done with the problem	65	92.8
2.Appropriateness of the steps or conditions specified in the instructions used in the problem	60	85.7
3.The amount of data contained in the problem in order to be able to solve the problem	64	91.4

The analysis of the problems posed by the pre-service mathematics teachers about square root expressions according to the instructions and the amount of data is given in Table 6. The maximum score that can be obtained according to this criterion is 210. The criterion of instructions and amount of data in the problems is the criterion with the highest success rate of 90%. In all three sub-criteria of this criterion, 60 and above points were obtained, and 85% success was achieved. It was concluded that most of the pre-service teachers did not use missing or excessive data in the problems they wrote and gave the necessary instructions to use the data correctly.

Table 7 – Analysis According to Solvability Criterion

Criteria and Sub-Criteria	Point	Percent
Solvability	51	72.8
1.Achievability of the desired outcome of the problem	51	72.8

The analysis of the problems related to square root expressions according to the solvability criterion is given in Table 6. According to Table 7, it was determined that 51 of the 70 problems were solvable and 29 were unsolvable problems and 72.8% success was achieved. Problems in which mathematical concepts were used incorrectly, the relationship between the visual and the text could not be established, and the instructions or data were incorrect were not deemed appropriate in terms of solvability criterion.

Table 8 – Analysis According to the Overall Assessment (Availability) Criteria

Criteria and Sub-Criteria	Point	Percent
General Assessment (Availability status)	45	64
1.The usability of the problem in the teaching process at the middle school level	45	64

The analysis of the posed problems according to the availability status is given in Table 8. Accordingly, it was determined that 45 of the 70 problems were usable and the success rate was found to be 64%. It has been observed that many of the solvable problems that have been established can be used in the teaching process, and many of the unsolvable problems are problems that can be made available in the teaching process by making the necessary corrections and eliminating deficiencies.

Discussion and Conclusion

In this study, it was aimed to examine pre-service mathematics teachers' problem posing skills related to square root expressions and the quality of the problems they posed. It was determined that 35 pre-service mathematics teachers who participated in the study posed a total of 70 problems using square root expressions and these problems were analyzed according to the problem posed evaluation criteria. When the sub-criteria of the mathematicality criterion were examined, it was found that they generally used mathematical expressions correctly, but for example, in a problem they wrote, they used the side length as "5" directly instead of 5 cm, which also shows that there are deficiencies in the expressions. In

addition, it was determined that they used some concepts such as the concept of "figure" incorrectly other than the meaning it encompasses, thus there were errors in the use of mathematical concepts. A word and a concept are not exactly the same thing (Kaşıkçı & Narlı, 2022). In this sense, the numbers written by teachers and the unit numbers they use mathematically may mean different things. In addition, it is known that there is an important relationship between problem posing skills and mathematical knowledge, and the extent to which mathematical concepts are understood can be interpreted based on the problems posed (Van-Harpen & Presmeg, 2013; Stoyanov, 2003). Based on this, it reveals the idea that the deficiencies in pre-service teachers' conceptual knowledge may have a negative effect on their problem posing skills (Demirci, 2018). These results overlap with the findings of Işık (2011), Kanbur (2017), Yıldız and Özdemir (2015). The symbol used to represent square root expressions was mostly used correctly. However, if there was any visual in the problem, it was observed that they made errors and deficiencies between the visual and the problem text. For example, they used a geometric figure in a problem they wrote, but no number in the problem statement was shown on this geometric figure and there was no connection between the figure and the problem.

When the problems about square root expressions were analyzed in terms of data quality, it was concluded that they contained logical and mathematical errors and deficiencies. For example, a pre-service teacher asked the area of a pencil box in the problem she wrote and the side lengths were written as $\sqrt{12}$ cm and $\sqrt{3}$ cm. In real life, there is no pencil box with these dimensions, so the data in the problem were not found to be logically appropriate. In another problem, the amount of money spent by a child is expressed as $4\sqrt{3}$ TL with roots. However, in daily life, we cannot express the amount of money as a square root and this was found to be logically incorrect. Since there were logic errors in the written problems, the result to be reached in the same problems was not considered meaningful. When analyzed in general, it was seen that a high level of success was not achieved regarding the data quality of the problems. It is also supported by many researchers that problem posing is an important tool in associating mathematics with real life and that such

problem posing activities improve mathematical thinking (Akay, 2006; Cunningham, 2004; Gözel, 2016). However, the fact that square root expressions are not used much in daily life and pre-service teachers' number sense skills in square root expressions are not sufficiently developed can be shown as a reason for them to construct problems containing mathematical logic errors. Likewise, Kanbur (2017) and Yıldız and Özdemir (2015) obtained findings supporting the results of the study.

Another criterion in which the problems related to square root expressions are analyzed is the correct use of grammar and expressions. It is noteworthy that the criterion in which pre-service mathematics teachers had the most difficulty and made the most mistakes was the correct use of grammar. It was observed that they made mistakes in the use of punctuation marks, spelling mistakes and expression disorder. As a result, it is seen that students have problems not only in using mathematical language but also in using Turkish. Students' inability to express and write down what they think affects problem posing to a great extent. Although grammar rules are related to the Turkish course and the correct use of Turkish, since we benefit from the language we speak when we express mathematics verbally, pre-service teachers should have sufficient knowledge about semantics, grammar and punctuation. As in every subject, having good language skills is the basis for understanding mathematics. Therefore, it is thought that teachers and pre-service teachers should pay attention to reading books as it is recommended to students.

The problems posed by pre-service mathematics teachers about square root expressions were analyzed in terms of another criterion, namely, instructions and amount of data. It was observed that there was generally no problem with the amount of data in the problems. Pre-service teachers mostly did not use missing data or unnecessary data in the problems. However, it was found that there were deficiencies in some problems about how to use the data in the problem. For example, in a problem in which square root expressions written in relation to probability were used, two bags were given side by side and there were square root expressions on these bags and the probability of these expressions being natural numbers when multiplied by each other was asked. However, it is not clear whether

these square root expressions should be multiplied by taking them from opposite bags or whether expressions in the same bag can be multiplied by each other. It is thought that the deficiencies in these instructions are also related to the previous criterion, so it is thought that pre-service teachers should improve themselves in understanding and explaining Turkish correctly. In addition, the inability to construct problems that contain sufficient and qualified data in problem posing may be due to the candidates' inadequacy in terms of mathematical content and conceptual aspects, as well as their unfamiliarity with problem posing skills.

Another criterion by which problems are examined is solvability. It cannot be said that the rate of solvable problems is very high. If there are conceptual errors in the problems, if there is no transfer between text and visuals, if there are logical errors in the data and if it is predicted that the result will not be meaningful, these problems are determined as unsolvable. The fact that square root expressions are not encountered much in daily life may have caused errors in making correct logical inferences about these numbers and transferring them to the problem.

When the problems are analyzed in general, it is seen that a little more than half of them can be used at the middle school level. The fact that this ratio is lower than the solvable problem ratio is due to the fact that problems at a higher level than the middle school level were written among the solvable problems. It was observed that pre-service teachers were able to construct problems related to square root expressions, but they had some difficulties in conceptual, logical and instructional aspects. Of course, this may be due to the fact that square root expressions are abstract and difficult by nature, but the lack of experience in problem posing is another reason for this. For this reason, it is recommended that studies on problem posing should be conducted especially in teacher training programs and in-service trainings.

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